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Complete Companion for JEE Main 2020 MATHEMATICS

Volume I

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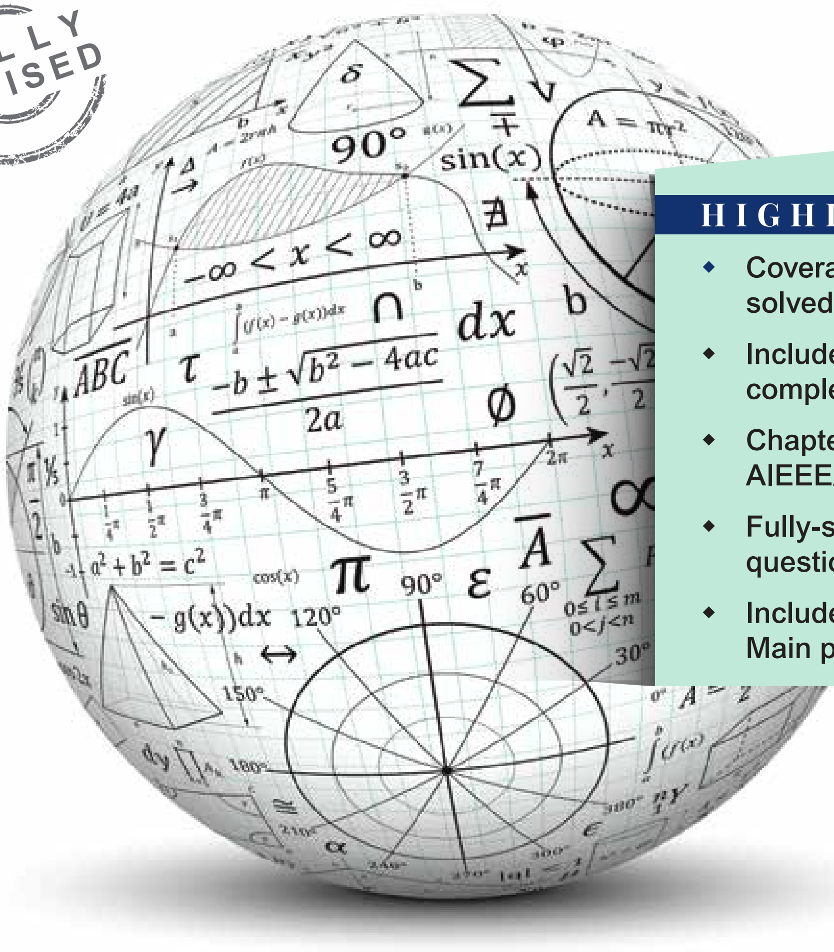

HIGHLIGHTS

- Coverage of key topics along with solved examples
- Includes practice problems with complete solutions
- Chapter-wise **Previous 18 years'** AIEEE/JEE Main questions
- Fully-solved **JEE Main 2019** questions (April & Jan)
- Includes **online tests** based on JEE Main pattern

FREE Online Mock Tests

Dinesh Khattar

Rohan Sinha

Pearson

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FIFTH EDITION

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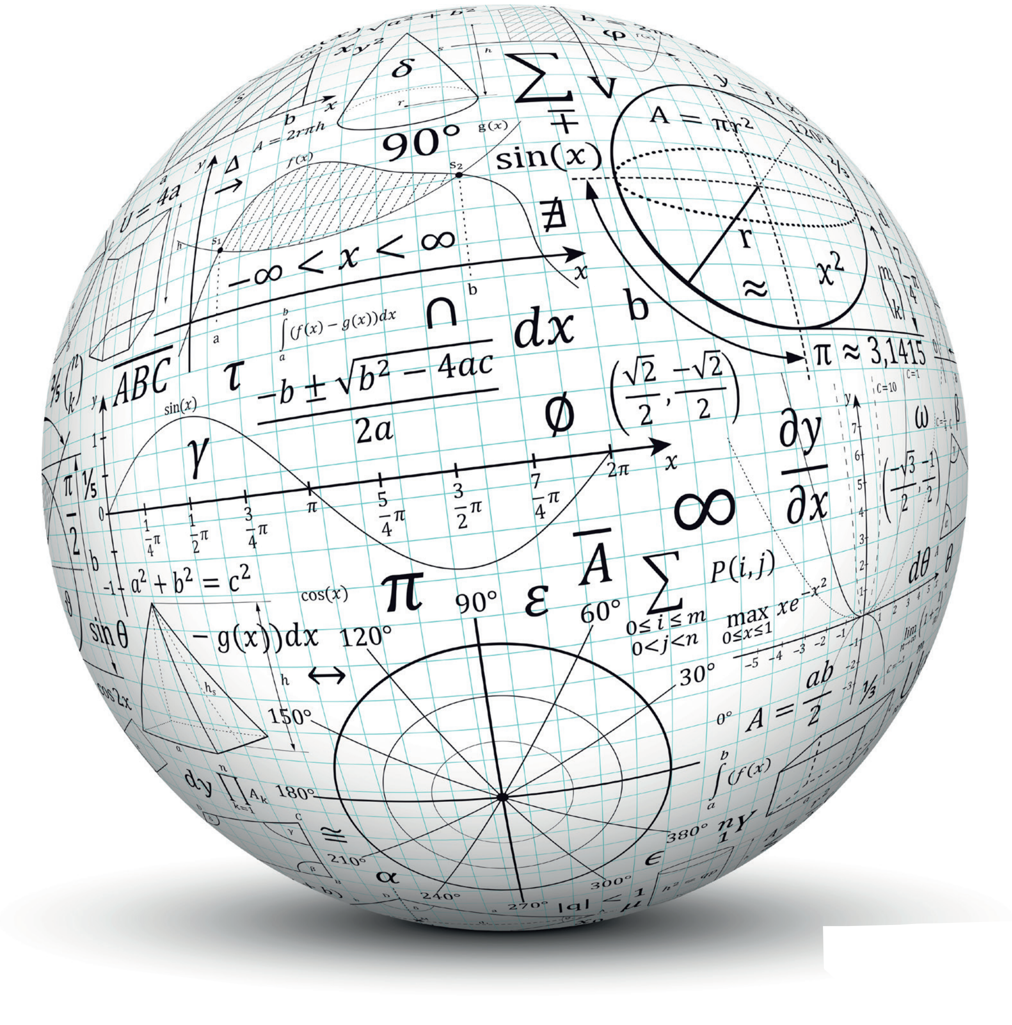
JEE Main 2020

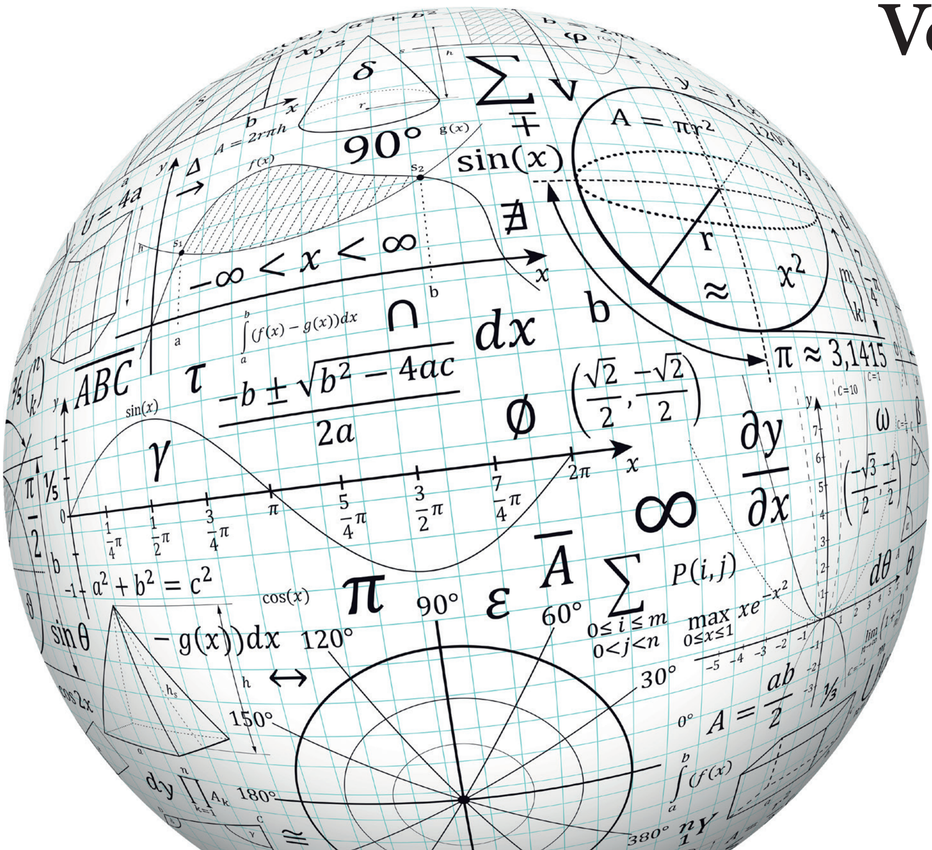
MATHEMATICS



FIFTH EDITION

Complete Companion for JEE Main 2020 MATHEMATICS

Volume I



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Preface

About the Series

Complete Companion for JEE Main series is a must have resource for students preparing for Joint Entrance Examination. There are three subjective books—Physics, Chemistry, and Mathematics; the core objective of this series is to strengthen the fundamental concepts and prepare students for various engineering entrance examinations. It provides class-tested course material and numerical applications that will supplement any ready material available as student resource.

To ensure high level of accuracy and practicality, this series has been authored by highly qualified and experienced faculties for all three titles.

About the Book

Complete Companion for JEE Main 2020 Mathematics, Volume 1 particularly developed for class 11th students, so that they can start their preparation from the early days. This title is designed as per the latest JEE Main syllabus, where the important topics are covered in 18 chapters. It has been structured in user friendly approach such that each chapter begins with topic-wise theory, followed by sufficient solved examples and then practice questions along with previous years' questions.

The chapter-end exercises are structured in line with JEE questions with ample of questions on single choice correct question (SCQ) for extensive practice. Previous 18 years' questions from JEE Main and AIEEE are also added in every chapter. Hints and Solutions at the end of every chapter will help the students to evaluate their concepts and numerical applications. Because of its comprehensive and in-depth approach, it will be especially helpful for those students who prefers self-study than going for any classroom teaching.

Series Features

- Complete coverage of topics along with ample number of solved examples
- Large variety of practice problems with complete solutions
- Chapter-wise Previous 18 years' AIEEE/JEE Main questions
- Fully solved JEE Main 2019 (Jan/Apr) questions added in opening section of the book
- Includes 5 Mock Tests papers based on JEE Main pattern in the book
- Free Online Mock Tests as per the recent JEE Main pattern

It would have been difficult to prepare this book without aid and support from a number of different quarters. I shall be grateful to the readers for their regular feedback. I am deeply indebted to my parents without whose encouragement this dream could not have been translated into reality. The cherubic smiles of my daughters, Nikita and Nishita, have inspired me to treat my work as worship.

Anuj Agarwal from IIT-Delhi, Ankit Katial from National Institute of Technology (Kurukshetra) and Raudrashish Chakraborty from Kirori Mal College, University of Delhi, with whom I have had fruitful discussions, deserve special mention.

I earnestly hope that the book will help the students grasp the subject well and respond with a commendable score in the JEE Main examination. There are a plethora of options available to students for Mathematics, however, ever grateful to them and to the readers for their candid feedback.

Despite of our best efforts, some errors may have crept into the book. Constructive comments and suggestions to further improve the book are welcome and shall be acknowledged gratefully.

Best of luck!

Dinesh Khattar

Mathematics Trend Analysis (2011 to 2019)

S. No.	Chapters	2011	2012	2013	2014	2015	2016	2017	2018	2019
Volume 1										
1	Sets	1	0	0	1	1	0	0	1	2
2	Complex Numbers and Quadratic Equations	7	2	10	3	0	5	3	1	4
3	Permutations and Combinations	0	3	2	2	2	2	1	1	2
4	Principles of Mathematical Induction	0	0	0	0	1	0	0	0	0
5	Binomial Theorem	0	0	2	1	1	2	1	1	3
6	Sequences and Series	2	1	3	3	4	3	0	2	1
7	Straight Iines	1	0	2	2	0	1	0	3	5
8	Conic Sections	5	3	2	6	0	6	0	7	4
9	Limits and Derivatives	2	4	2	8	11	5	4	2	3
10	Mathematical Reasoning	0	0	1	1	1	1	1	1	1
Volume 2										
11	Relations and Functions	3	2	1	0	1	1	1	1	2
12	Inverse Trigonometric Functions	1	0	2	0	2	0	1	1	1
13	Matrices	2	2	2	1	2	3	1	1	3
14	Determinants	2	1	2	1	2	3	1	1	2
15	Continuity and Differentiability	2	1	2	1	8	4	2	2	3
16	Applications of Derivatives	2	6	9	3	3	4	2	1	2
17	Integrals	2	2	2	3	3	4	1	2	3
18	Applications of Intergrals	1	1	2	2	3	3	1	1	2
19	Differential Equation	2	1	2	1	3	3	1	1	4
20	Vector Algebra	3	2	3	3	3	2	1	1	3
21	Three-dimensional Geometry	0	3	7	3	6	4	3	1	3
22	Probability	3	3	5	1	0	4	2	2	3

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CHAPTER 1

Set Theory

LEARNING OBJECTIVES

After reading this chapter, you will be able to:

- Learn the definition of set and how is it denoted
- Know how the sets are represented and what are its types
- Understand the operations on sets and Identify some key results
- Establish a relation between the two sets and study about its types

SET

A *set* is a well-defined collection of objects such that given an object, it is possible to determine whether that object belongs to the given collection or not.

For example, the collection of all students of Delhi University is a set, whereas, collection of all good books on mathematics is not a set, since a mathematics book considered good by one person might be considered bad or average by another.

Notations

The sets are usually denoted by capital letters A, B, C, etc. and the members or elements of the set are denoted by lower-case letters a, b, c etc. If x is a member of the set A, we write $x \in A$ (read as 'x belongs to A') and if x is not a member of the set A, we write $x \notin A$ (read as 'x does not belong to A'). If x and y both belong to A, we write $x, y \in A$.

REPRESENTATION OF A SET

Usually, sets are represented in the following two ways:

1. Roster form or tabular form
2. Set builder form or rule method

Roster Form

In this form, we list all the members of the set within braces (curly brackets) and separate these by commas.

For example, the set A of all odd natural numbers less than 10 in the roster form is written as:

$$A = \{1, 3, 5, 7, 9\}$$

IMPORTANT POINTS

- In roster form, every element of the set is listed only once.
- The order in which the elements are listed is immaterial For example, each of the following sets denotes the same set $\{1, 2, 3\}$, $\{3, 2, 1\}$, $\{1, 3, 2\}$.

Set-builder Form

In this form, we write a variable (say x) representing any member of the set followed by a property satisfied by each member of the set.

For example, the set A of all prime numbers less than 10 in the set-builder form is written as

$$A = \{x \mid x \text{ is a prime number less than } 10\}$$

The symbol '|' stands for the words 'such that'. Sometimes, we use the symbol ':' in place of the symbol '|'.

TYPES OF SETS

Empty Set or Null Set

A set which has no element is called the *null set* or *empty set*. It is denoted by the symbol ϕ.

For example, each of the following is a null set:

1. The set of all real numbers whose square is -1.
2. The set of all rational numbers whose square is 2.
3. The set of all those integers that are both even and odd.

A set consisting of atleast one element is called a *non-empty set*.

Singleton Set

A set having only one element is called *singleton set*.

For example, $\{0\}$ is a singleton set, whose only member is 0.

Finite and Infinite Set

A set which has finite number of elements is called a *finite set*. Otherwise, it is called an *infinite set*.

For example, the set of all days in a week is a finite set whereas, the set of all integers, denoted by $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ or $\{x \mid x \text{ is an integer}\}$, is an infinite set.

An empty set ϕ which has no element, is a finite set.

The number of distinct elements in a finite set A is called the *cardinal number of the set A* and it is denoted by $n\,(A)$.

Equal Sets

Two sets A and B are said to be *equal*, written as $A = B$, if every element of A is in B and every element of B is in A.

Equivalent Sets

Two finite sets A and B are said to be *equivalent*, if $n(A) = n(B)$.

ERROR CHECK

Equal sets are equivalent but equivalent sets need not be equal.

For example, the sets $A = \{4, 5, 3, 2\}$ and $B = \{1, 6, 8, 9\}$ are equivalent but are not equal.

Subset

Let A and B be two sets. If every element of A is an element of B, then A is called a *subset* of B and we write $A \subseteq B$ or $B \supseteq A$ (read as 'A is contained in B' or B contains A'). B is called *superset* of A.

IMPORTANT POINTS

- If $A \subseteq B$ and $A \neq B$, we write $A \subset B$ or $B \supset A$ (read as : A is a proper subset of B or B is a proper superset of A).
- Every set is a subset and a superset of itself.
- If A is not a subset of B, we write $A \not\subset B$.
- The empty set is the subset of every set.
- If A is a set with $n\,(A) = m$, then the number of subsets of A are 2^m and the number of proper subsets of A are $2^m - 1$.

For example, let $A = \{3, 4\}$, then the subsets of A are ϕ, $\{3\}$, $\{4\}$, $\{3, 4\}$. Here, $n(A) = 2$ and number of subsets of $A = 2^2 = 4$.

Also, $\{3\} \subset \{3, 4\}$ and $\{2, 3\} \not\subseteq \{3, 4\}$

Power Set

The set of all subsets of a given set A is called the *power set* of A and is denoted by $P(A)$.

For example, if $A = \{1, 2, 3\}$, then

$$P(A) = [\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}\ \{1, 3\}, \{2, 3\}, \{1, 2, 3\}]$$

Clearly, if A has n elements, then its power set $P(A)$ contains exactly 2^n elements.

QUICK TIPS

Number of elements in $P\,\{P\,[P(\phi)]\}$ is 4

or Cardinal Number of $P\{P[P(\phi)]\} = 4$

Since, $P\,(\phi) = \{\phi\}$

Also, $P\,[P\,(\phi)] = \{\phi, \{\phi\}\}$

and $P\,\{P[P\,(\phi)]\} = \{\phi, \{\phi\}, [\{\phi\}], [\phi, \{\phi\}]\}$

Hence, $n\,[P\,\{P[P\,(\phi)]\}] = 4$

Euler–Venn Diagrams

To express the relationship among sets, we represent them pictorially by means of diagrams, known as Euler–Venn Diagrams or simply Venn diagrams.

In Venn diagrams, the universal set U is represented by the rectangular region and its subsets are represented by closed bounded circles inside this rectangular region.

OPERATIONS ON SETS

Union of Two Sets

The union of two sets A and B, written as $A \cup B$ (read as 'A union B'), is the set consisting of all the elements which are either in A or in B or in both. Thus,

$$A \cup B = \{x : x \in A \quad \text{or} \quad x \in B\}$$

Clearly, $x \in A \cup B$

$\Rightarrow$ $x \in A$

or $x \in B$,

and $x \notin A \cup B$

$\Rightarrow$ $x \notin A$

and $x \notin B$.

For example, if $A = \{a, b, c\ d\}$ and $B = \{c, d, e, f\}$, then $A \cup B = \{a, b, c, d, e, f\}$.

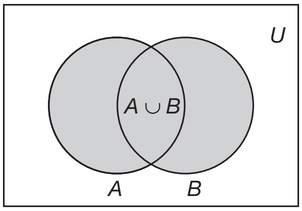


FIGURE 1.1

Intersection of Two Sets

The intersection of two sets A and B, written as $A \cap B$ (read as 'A intersection B') is the set consisting of all the common elements of A and B. Thus,

$$A \cap B = \{x: x \in A \text{ and } x \in B\}$$

Clearly, $\quad x \in A \cap B$

$\Rightarrow \quad x \in A \quad \text{and} \quad x \in B,$

and $\quad x \notin A \cap B$

$\Rightarrow \quad x \notin A$ or $x \notin B$.

For example, if $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$, then $A \cap B = \{c, d\}$.

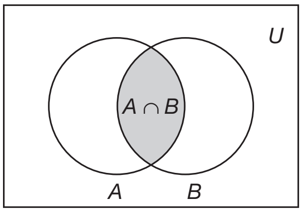


FIGURE 1.2

Disjoint Sets

Two sets A and B are said to be *disjoint*, if $A \cap B = \phi$, i.e., A and B have no element in common.

For example, if $A = \{1, 2, 5\}$ and $B = \{2, 4, 6\}$, then $A \cap B = \phi$, so A and B are disjoint sets.

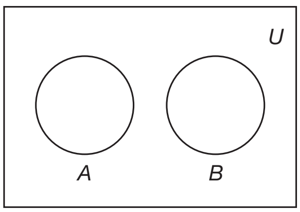


FIGURE 1.3

Difference of Two Sets

If A and B are two sets, then their difference $A - B$ is defined as

$$A - B = \{x: x \in A \text{ and } x \notin B\}$$

Similarly,

$$B - A = \{x: x \in B \text{ and } x \notin A\}$$

For example,

if $\quad A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7, 9\}$,

then $\quad A - B = \{2, 4\}$ and $B - A = \{7, 9\}$

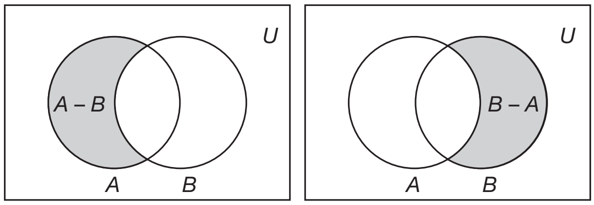


FIGURE 1.4 (a–b)

Info Box!

- $A - B \neq B - A$
- The sets $A - B$, $B - A$ and $A \cap B$ are disjoint sets
- $A - B \subseteq A$ and $B - A \subseteq B$
- $A - \phi = A$ and $A - A = \phi$

Symmetric Difference of Two Sets

The symmetric difference of two sets A and B, denoted by $A \Delta B$, is defined as

$$A \Delta B = (A - B) \cup (B - A).$$

For example, if $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7, 9\}$ then $A \Delta B = (A - B) \cup (B - A) = \{2, 4\} \cup \{7, 9\} = \{2, 4, 7, 9\}$.

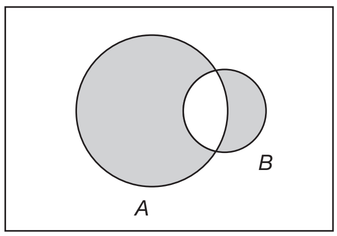


FIGURE 1.5

Complement of a Set

If U is a universal set and A is a subset of U, then the complement of A is the set which contains those elements of U, which are not contained in A and is denoted by A' or A^c. Thus,

$$A' = \{x: x \in U \text{ and } x \notin A\}$$

For example,

if $U = \{1, 2, 3, 4, \ldots\}$ and $A = \{2, 4, 6, 8, \ldots\}$,

then, $A' = \{1, 3, 5, 7, \ldots\}$

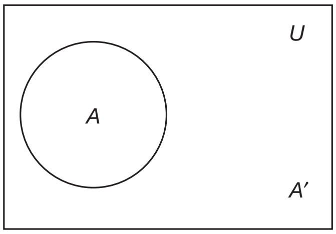


FIGURE 1.6

Info Box!

- $U' = \phi$
- $\phi' = U$
- $A \cup A' = U$
- $A \cap A' = \phi$

ALGEBRA OF SETS

1. **Idempotent Laws:** For any set A, we have
   (a) $A \cup A = A$
   (b) $A \cap A = A$
2. **Identity Laws:** For any set A, we have:
   (a) $A \cup \phi = A$
   (b) $A \cap \phi = \phi$
   (c) $A \cup U = U$
   (d) $A \cap U = A$
3. **Commutative Laws:** For any two sets A and B, we have
   (a) $A \cup B = B \cup A$
   (b) $A \cap B = B \cap A$
4. **Associative Laws:** For any three sets A, B and C, we have
   (a) $A \cup (B \cup C) = (A \cup B) \cup C$
   (b) $A \cap (B \cap C) = (A \cap B) \cap C$
5. **Distributive Laws:** For any three sets A, B and C, we have
   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
6. For any two sets A and B, we have
   (a) $P(A) \cap P(B) = P(A \cap B)$
   (b) $P(A) \cup P(B) \subseteq P(A \cup B)$, where $P(A)$ is the power set of A.
7. If A is any set, we have $(A')' = A$.
8. **Demorgan's Laws:** For any three sets A, B and C, we have
   (a) $(A \cup B)' = A' \cap B'$
   (b) $(A \cap B)' = A' \cup B'$
   (c) $A - (B \cup C) = (A - B) \cap (A - C)$
   (d) $A - (B \cap C) = (A - B) \cup (A - C)$

Key Results on Operations on Sets

1. $A \subseteq A \cup B$, $B \subseteq A \cup B$, $A \cap B \subseteq A$, $A \cap B \subseteq B$
2. $A - B = A \cap B'$
3. $(A - B) \cup B = A \cup B$
4. $(A - B) \cap B = \phi$
5. $A \subseteq B \Leftrightarrow B' \subseteq A'$
6. $A - B = B' - A'$
7. $(A \cup B) \cap (A \cup B') = A$
8. $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$
9. $A - (A - B) = A \cap B$
10. $A - B = B - A \Leftrightarrow A = B$
11. $A \cup B = A \cap B \Leftrightarrow A = B$
12. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Some Results about Cardinal Number

If A, B and C are finite sets and U be the finite universal set, then

1. $n(A') = n(U) - n(A)$
2. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
3. $n(A \cup B) = n(A) + n(B)$,
   where A and B are disjoint non-empty sets
4. $n(A \cap B') = n(A) - n(A \cap B)$
5. $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
6. $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
7. $n(A - B) = n(A) - n(A \cap B)$
8. $n(A \cap B) = n(A \cup B) - n(A \cap B') - n(A' \cap B)$
9. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
10. If $A_1, A_2, A_3, \ldots A_n$ are disjoint sets, then
    $n(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n) = n(A_1) + n(A_2) + n(A_3) + \ldots + n(A_n)$
11. $n(A \Delta B)$ = number of elements which belong to exactly one of A or B

CARTESIAN PRODUCT OF TWO SETS

If A and B are any two non-empty sets, then *cartesian product* of A and B is defined as

$$A \times B = [(a, b) : a \in A \text{ and } b \in B]$$

ERROR CHECK

$A \times B \neq B \times A$

QUICK TIPS

- If $A = \phi$ or $B = \phi$, then we define $A \times B = \phi$.
- If A has n elements and B has m elements then $A \times B$ has mn elements.
- If $A_1, A_2, \ldots, A_p$ are p non-empty sets, then their cartesian product, is defined as $\prod_{i=1}^{p} A_i = [(a_1, a_2, a_3, \ldots, a_p);\ a_i \in A_i$ for all $i]$

Key Results on Cartesian Product

If A, B, C are three sets, then

1. $A \times (B \cup C) = (A \times B) \cup (A \times C)$
2. $A \times (B \cap C) = (A \times B) \cap (A \times C)$
3. $A \times (B - C) = (A \times B) - (A \times C)$
4. $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$, where S and T are two sets.
5. If $A \subseteq B$, then $(A \times C) \subseteq (B \times C)$
6. If $A \subseteq B$, then $(A \times B) \cap (B \times A) = A^2$
7. If $A \subseteq B$ and $C \subseteq D$ then $A \times C \subseteq B \times D$
8. If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$
9. If A and B are two non-empty sets having n elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.
10. $A \times B = B \times A$ if and only if $A = B$
11. $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$
12. $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

SOLVED EXAMPLES

1. If $n(U) = 60$, $n(A) = 35$, $n(B) = 24$ and $n(A \cup B)' = 10$ then $n(A \cap B)$ is

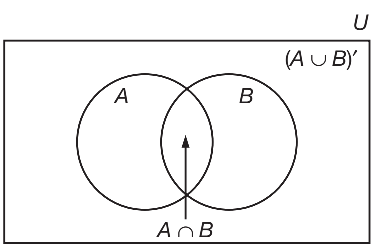
(A) 9 (B) 8
(C) 6 (D) None of these

Solution: (A)

We have,

$$n(A \cup B) = n(U) - n(A \cup B)' = 60 - 10 = 50$$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



$$\Rightarrow \quad 50 = 35 + 24 - n(A \cap B)$$

$$\Rightarrow \quad n(A \cap B) = 59 - 50 = 9.$$

2. Let $A = \{2, 3, 4\}$ and $X = \{0, 1, 2, 3, 4\}$, then which of the following statements is correct?

(A) $\{0\} \in A'$ in X
(B) $\phi \in A'$ with respect to. X
(C) $\{0\} \subset A'$ with respect to X
(D) $0 \subset A'$ with respect to X.

Solution: (C)

We have, A' in X = The set of elements in X which are not in $A = \{0, 1\}$

$\{0\} \in A'$ in X is false, because $\{0\}$ is not an element of A' in X.

$\phi \subset A'$ in X is false, because ϕ is not an element of A' in X

$\{0\} \subset A'$ in X is correct, because the only element of $\{0\}$ namely 0 also belongs to A' in X.

$0 \subset A'$ in X is false, because 0 is not a set.

3. If $X = \{8^n - 7n - 1 / n \in N\}$ and $Y = \{49(n-1) / n \in N\}$, then

(A) $X \subset Y$ (B) $Y \subset X$
(C) $X = Y$ (D) None of these

Solution: (A)

We have, $8^n - 7n - 1$

$$= (7+1)^n - 7n - 1 = ({}^nC_2 7^2 + {}^nC_3 7^3 + \ldots + {}^nC_n 7^n)$$

$$= 49({}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_n 7^{n-2}) \text{ for } n \ge 2$$

For $n = 1$, $8^n - 7n - 1 = 0$

Thus, $8^n - 7n - 1$ is a multiple of 49 for $n \ge 2$ and 0 for $n = 1$. Hence, X consists of all positive integral multiples of 49 of the form $49 K_n$, where $K_n = {}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_n 7^{n-2}$ together with zero. Also, Y consists of all positive integral multiples of 49 including zero. Therefore, $X \subset Y$.

4. The set $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to

(A) $A \cap B$ (B) $A \cap C'$
(C) $B \cap C'$ (D) $B' \cap C'$

Solution: (C)

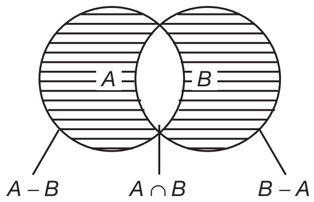
$$(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$$
$$= (A \cup B \cup C) \cap (A' \cup B \cup C) \cap C'$$
$$= [(A \cap A') \cup (B \cup C)] \cap C'$$
$$= (\phi \cup B \cup C) \cap C' = (B \cup C) \cap C'$$
$$= (B \cap C') \cup (C \cap C')$$
$$= (B \cap C') \cup \phi = B \cap C'$$

5. If A, B and C are non-empty subsets of a set, then $(A - B) \cup (B - A)$ equals

(A) $(A \cap B) \cup (A \cup B)$ (B) $(A \cup B) - (A \cap B)$
(C) $A - (A \cap B)$ (D) $(A \cup B) - B$

Solution: (B)

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$



6. Let A and B two non-empty subsets of a set X such that A is not a subset of B then

(A) A is subset of the complement of B
(C) B is a subset of A
(C) A and B are disjoint
(D) A and the complement of B are non-disjoint

Solution: (D)

Since $A \not\subseteq B$, $\exists\, x \in A$ such that $x \notin B$

Then $x \in B'$.

$\therefore$ $A \cap B' \neq \phi$

7. Two finite sets have m and n elements, then total number of subsets of the first set is 56 more that the total number of subsets of the second. The values of m and n are,

(A) 7, 6 (B) 6, 3 (D) 5, 1 (D) 8, 7

Solution: (B)

Since the two finite sets have m and n elements, so number of subsets of these sets will be 2^m and 2^n respectively. According to the question

$$2^m - 2^n = 56$$

putting $m = 6$, $n = 3$, we get

$$2^6 - 2^3 = 56 \quad \text{or} \quad 64 - 8 = 56$$

8. Let $U = R$ (the set of all real numbers) If $A = \{x : x \in R, 0 < x < 2\}$, $B = \{x : x \in R, 1 < x \le 3\}$, then

(A) $A \cup B = \{x : x \in R \text{ and } 0 < x \le 3\}$
(B) $A \cap B = \{x : x \in R \text{ and } 1 < x < 2\}$
(C) $A - B = \{x : x \in R \text{ and } 0 < x \le 1\}$
(D) All of these

Solution: (D)

We have

$$A' = R - A = \{x : \in R \text{ and } x \notin A\}$$

$$= \{x : (x \in R \text{ and } x \ge 2) \text{ or } (x \in R \text{ and } x \le 0)\}$$

$$= \{x : x \in R \text{ and } x \ge 2\} \cup \{x : x \in R \text{ and } x \le 0\}$$

Similarly,

$$B' = \{x : x \in R \text{ and } x \le 1\} \cup \{x : x \in R \text{ and } x > 3\}$$

$\therefore$ $A \cup B = \{x : x \in R \text{ and } 0 < x \le 3\}$,

$A \cap B = \{x : x \in R \text{ and } 1 < x < 2\}$

$A - B = \{x : x \in R \text{ and } 0 < x \le 1\}$

9. If $Y \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$, then

(A) The smallest set of Y is $\{3, 5, 9\}$
(B) The smallest set of Y is $\{2, 3, 5, 9\}$
(C) The largest set of Y is $\{1, 2, 3, 4, 9\}$
(D) The largest set of Y is $\{2, 3, 4, 9\}$

Solution: (A and C)

Since the set on the right hand side has 5 elements,

$\therefore$ smallest set of Y has three elements and largest set of Y has five elements,

$\therefore$ smallest set of Y is $\{3, 5, 9\}$

and largest set of Y is $\{1, 2, 3, 4, 9\}$.

10. If A has 3 elements and B has 6 elements, then the minimum number of elements in the set $A \cup B$ is

(A) 6 (B) 3
(C) ϕ (D) None of these

Solution: (A)

Clearly the number of elements in $A \cup B$ will be minimum when $A \subset B$. Hence the minimum number of elements in $A \cup B$ is the same as the number of elements in B, that is, 6.

11. Suppose $A_1, A_2, \ldots A_{30}$ are thirty sets, each with five elements and $B_1, B_2, \ldots, B_n$ are n sets each with three elements. Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$

If each element of S belongs to exactly ten of the A_i's and exactly nine of the B_j's then $n =$

(A) 45 (B) 35
(C) 40 (D) None of these

Solution: (A)

Given A_i's are thirty sets with five elements each, so

$$\sum_{i=1}^{30} n(A_i) = 5 \times 30 = 150 \quad (1)$$

If there are m distinct elements in S and each element of S belongs to exactly 10 of the A_i's, we have

$$\sum_{i=1}^{30} n(A_i) = 10\text{ m} \quad (2)$$

∴ From Eq. (1) and (2), we get

$$10\,m = 150$$

$$\therefore \quad m = 15 \quad (3)$$

Similarly $\sum_{j=1}^{30} n(B_j) = 3n$ and $\sum_{j=1}^{30} n(B_j) = 9\,m$

$$\therefore \quad 3\,n = 9\,m$$

$$\Rightarrow \quad n = \frac{9m}{3} = 3\,m = 3 \times 15 = 45 \quad \text{[from (3)]}$$

Hence, $n = 45$.

12. If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, \ldots, 18\}$ and N is the universal set, then $A' \cup ((A \cup B) \cap B')$ is

(A) A (B) N
(C) B (D) None of these

Solution: (B)

We have,

$$(A \cup B) \cap B' = A$$

$$[(A \cup B) \cap B'] \cup A' = A \cup A' = N.$$

13. If X and Y are two sets and X' denotes the complement of X, then $X \cap (X \cup Y)'$ equals

(A) X (B) Y
(C) ϕ (D) None of these

Solution: (C)

$$X \cap (X \cup Y)' = X \cap (X' \cap Y')$$

[∵ By De-Morgan's Law $(A \cup B)' = (A' \cap B')$]

$$= (X \cap X') \cap Y' = \phi \cap Y' = \phi$$

14. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. The number of persons liking tennis only and not cricket is

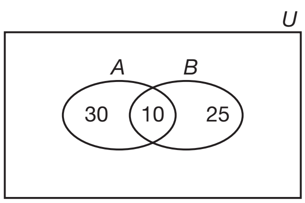
(A) 21 (B) 25
(C) 15 (D) None of these

Solution: (B)

Let A be the set of people who like cricket and B the set of people who like tennis.

Then $n(A \cup B) = 65$

$$n(A) = 40,\ n(A \cap B) = 10$$



$$\because \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$65 = 40 + n(B) - 10$$

$$n(B) = 65 - 40 + 10 = 35$$

Number of people who like only tennis

$$= n(B) - n(A \cap B) = 35 - 10 = 25$$

∴ Number of people who like tennis only and not cricket = 25.

15. In a group of 1000 people, there are 750 people who can speak Hindi and 400 who can speak English. Then number of persons who can speak Hindi only is

(A) 300 (B) 400
(C) 600 (D) None of these

Solution: (C)

Here

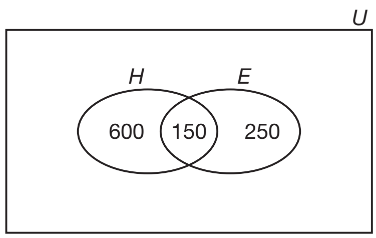
$$n(H \cup E) = 1000,\ n(H) = 750,$$

$$n(E) = 400$$

Using $n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$$1000 = 750 + 400 - n(H \cap E)$$

$$\Rightarrow \quad n(H \cap E) = 1150 - 1000 = 150.$$



Number of people who can speak Hindi only

$$= n(H \cap E') = n(H) - n(H \cap E)$$

$$= 750 - 150 = 600.$$

16. If $f: R \to R$, defined by $f(x) = x^2 + 1$, then the values of $f^{-1}(17)$ and $f^{-1}(-3)$ respectively are

(A) ϕ, {4, –4} (B) {3, –3}, ϕ
(C) ϕ, {3, –3} (D) {4, –4}, ϕ

Solution: (D)

Let $y = x^2 + 1$. Then for $y = 17$,

we have $x = \pm 4$ and for $y = -3$, x becomes imaginary that is, there is no value of x.

Hence, $f(17) = \{-4, 4\}$

and $f^{1}(-3) = \phi$

17. In a statistical investigation of 1,003 families of Kolkata, it was found that 63 families had neither a radio nor a TV, 794 families had a radio and 187 had a TV. The number of families in that group having both a radio and a TV is

(A) 36 (B) 41
(C) 32 (D) None of these

Solution: (B)

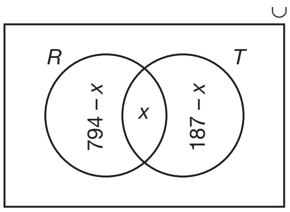
Let R be the set of families having a radio and T, the set of families having a TV, then

$n(R \cup T)$ = The no. of families having at least one of radio and TV

$$= 1003 - 63 = 940$$

$$n(R) = 794 \text{ and } n(T) = 187$$

Let x families had both a radio and a TV i.e.,



$$n(R \cap T) = x$$

The number of families who have only radio = $794 - x$ and the number of families who have only TV = $187 - x$

From Venn diagram,

$$794 - x + x + 187 - x = 940$$

$$\Rightarrow \quad 981 - x = 940 \text{ or } x = 981 - 940 = 41$$

Hence, the required no. of families having both a radio and a TV = 41.

18. In a city, three daily newspapers A, B, C are published. 42% of the people in that city read A, 51% read B and 68% read C. 30% read A and B; 28% read B and C; 36% read A and C; 8% do not read any of the three newspapers. The percentage of persons who read all the three papers is

(A) 25% (B) 18%
(C) 20% (D) None of these

Solution: (A)

Let the no. of persons in the city be 100.

Then we have

$$n(A) = 42,\ n(B) = 51,\ n(C) = 68;$$

$$n(A \cap B) = 30,\ n(B \cap C) = 28,\ n(A \cap C) = 36$$

$$n(A \cup B \cup C) = 100 - 8 = 92$$

Using

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$$
$$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Substituting the above values, we have

$$92 = 42 + 51 + 68 - 30 - 28 - 36 + n(A \cap B \cap C)$$

$$\Rightarrow \quad n(A \cap B \cap C) = 92 - 161 + 94$$

$$\Rightarrow \quad n(A \cap B \cap C) = 92 - 67 = 25$$

Hence, 25% of the people read all the three papers.

19. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is

(A) 160 (B) 240 (C) 216 (D) 128

Solution: (A)

$$n(C) = 224,\ n(H) = 240,\ n(B) = 336$$

$$n(H \cap B) = 64,\ n(B \cap C) = 80$$

$$n(H \cap C) = 40,\ n(C \cap H \cap B) = 24$$

$$n(C^c \cap H^c \cap B^c) = n[(C \cup H \cup B)^c]$$
$$= n(U) - n(C \cup H \cup B)$$
$$= 800 - [n(C) + n(H) + n(B) - n(H \cap C)$$
$$- n(H \cap B) - n(C \cap B) + n(C \cap H \cap B)]$$
$$= 800 - [224 + 240 + 336 - 64 - 80 - 40 + 24]$$
$$= 800 - [824 - 184] = 984 - 824 = 160.$$

20. In a certain town 25% families own a phone and 15% own a car, 65% families own neither a phone nor a car. 2000 families own both a car and a phone. Consider the following statements in this regard:

1. 10% families own both a car and a phone.
2. 35% families own either a car or a phone.
3. 40,000 families live in the town.

Which of the above statements are correct?

(A) 1 and 2 (B) 1 and 3
(C) 2 and 3 (D) 1, 2 and 3

Solution: (C)

$$n(P) = 25\%,\ n(C) = 15\%,$$

$$n(P' \cap C') = 65\%,\ n(P \cap C) = 2000$$

Since, $\quad n(P' \cap C') = 65\%$

$\therefore \quad n(P \cup C)' = 65\%$

$\therefore \quad n(P \cup C) = 35\%$

Now, $\quad n(P \cup C) = n(P) + n(C) - n(P \cap C)$

$\therefore \quad 35 = 25 + 15 - n(P \cap C)$

$\therefore \quad n(P \cap C) = 40 - 35 = 5$

Thus, $\quad n(P \cap C) = 5\%$

But, $\quad n(P \cap C) = 2000$

$\therefore$ 5% of the total = 2000

$\therefore$ total no. of families $= \dfrac{2000 \times 100}{5} = 40000$

$\therefore \quad n(P \cup C) = 35\%$,

Total no. of families = 40,000 and $n(P \cap C) = 5\%$.

21. If P, Q and R are subsets of a set A, then $R \times (P' \cup Q')'$ equals

(A) $(R \times P) \cap (R \times Q)$ (B) $(R \times Q) \cap (R \times P)$
(C) $(R \times P) \cup (R \times Q)$ (D) None of these

Solution: (A)

$$R \times (P' \cup Q')' = R \times [(P')' \cap (Q')']$$
$$= R \times (P \cap Q)$$
$$= (R \times P) \cap (R \times Q)$$

22. If sets A and B are defined as

$A = \{(x, y) : y = e^x, x \in R\}$
$B = \{(x, y) : y = x, x \in R\}$ then

(A) $B \subset A$ (B) $A \subset B$
(C) $A \cap B = \phi$ (D) $A \cup B = A$

Solution: (C)

Since $\quad y = e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$

$\therefore e^x > x \ \forall \ x \in R$ so that the two curves given by $y = e^x$ and $y = x$ do not intersect in any point Hence, there is no common point, so that $A \cap B = \phi$.

23. The solution of $3x^2 - 12x = 0$ when

(A) $x \in N$ is $\{4\}$
(B) $x \in I$ is $\{0, 4\}$
(C) $x \in S = \{a + ib : b \neq 0, a, b \in R\}$ is ϕ
(D) All of these

Solution: (D)

We have,

$$3x^2 - 12x = 0$$
$$\Rightarrow \quad 3x(x - 4) = 0$$
$$\Rightarrow \quad x = 0, 4$$

Now, if $x \in N$, then the solution set is $\{4\}$.

Also, if $x \in I$, then the solution set is $\{0, 4\}$.

Further, since there is no root of the form $a + ib$, where a, b are real and $b \neq 0$,

$\therefore$ if $x \in S = \{a + ib : b \neq 0, a, b \in R\}$ then the solution set is ϕ.

RELATIONS

Let A, B be any two non-empty sets, then every subset of $A \times B$ defines a relation from A to B and every relation from A to B is a subset of $A \times B$.

If R is a relation from A to B and if $(a, b) \in R$, then we write $a \, R \, b$ and say that 'a is related to b' and if $(a, b) \notin R$, then we write $a \not R \, b$ and say that a is not related to b.

Key Results on Relations

1. Every subset of $A \times A$ is said to be a relation on A.
2. If A has m elements and B has n elements, then $A \times B$ has mn elements and total number of different relations from A to B is 2^{mn}.
3. Let R be a relation from A to B, *i.e.* $R \subseteq A \times B$, then
   Domain of $R = \{a : a \in A, (a, b) \in R$ for some $b \in B\}$
   Range of $R = \{b : b \in B, (a, b) \in R$ for some $a \in A\}$
   For example, let $A = \{1, 3, 4, 5, 7\}$, $B = \{2, 4, 6, 8\}$ and R be the relation 'is one less than' from A to B, then $R = [(1, 2), (3, 4), (5, 6), (7, 8)]$.
   Here, domain of $R = \{1, 3, 5, 7\}$ and range of $R = \{2, 4, 6, 8\}$.

Info Box!

Domain of a relation from A to B is a subset of A and its range is a subset of B

Identity Relation

R is an *identity relation* if $(a, b) \in R$ if $a = b$, $a \in A$, $b \in A$. In other words, every element of A is related to only itself.

Universal Relation

Let A be any set and R be the set $A \times A$, then R is called the *universal relation* in A.

Void Relation

ϕ is called void relation in a set.

QUICK TIPS

The void and the universal relations on a set A are respectively the smallest and the largest relations on A